FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 9 No. 11, November, 2025, pp 345 – 355
DOI: https://doi.org/10.33003/fjs-2025-0911-4189

UNSTEADY MAGNETOHYDRODYNAMICS (MHD) FLOW OF FOURTH GRADE FLUID BETWEEN HORIZONTAL STATIONARY PLATES WITH VISCOUS DISSIPATION AND SUCTION EFFECTS

***[1,2]Nasiru T. Zakari, [2]Ayankop E. Andi, [3]Joseph K. Moses, [4]Wachin, A. A., [5]Mundi. I. Baba and [5]Joshua B. Hassan**

[1]Department of Mathematical Sciences, Nigerian Defence Academy, Kaduna, Nigeria
[2]Department of Mathematical Sciences, Kaduna State University, Kaduna, Nigeria
[3]Department of mathematics, Air force Institute of Technology, Kaduna, Nigeria
[4]Department of Mathematics, Nigerian Institute of Transport Technology, Zaria, Nigeria
[5]Department of Computer Science, Federal University Oye, Ekiti, Nigeria

*Corresponding authors' email: ntzakari@gmail.com

ABSTRACT

The unsteady magnetohydrodynamics (MHD) flow of fourth grade fluid between horizontal stationary plates with viscous dissipation and suction effects has been investigated. The fluid is subjected to a uniform transverse magnetic field. The upper and lower plates are stationary. The partial differential equations that govern the flow are the momentum and energy equations. For the solution of the fluid flow model, the He- Laplace method was employed. The effect of various flow parameters on velocity and temperature profile were obtained. Investigated were the effects of suction parameter, third and fourth grade parameters and Hartman number on velocity profile. Graph were plotted for each case considered. Results of this research show that temperature field increases with increase in Eckert number and radiation parameter but decrease with increases in suction parameter and Prandtl number. While for velocity field, it is observed that it decreases with increase in suction parameter and Hartman number, but increases with increase in third and fourth grade parameters. The result of this research would be useful in Engineering fields such as Bioengineering enhance oil recovery and medical science of complex fluid in development of magnetic device for cell separation and targeted transport of drugs carriers, etc.

Keywords: MHD, Unsteady, Viscous dissipation, Suction, Fourth-grade fluid

INTRODUCTION

The fourth-grade fluid flow model is an exceptional model which has opened new subway of fluid mechanics. This sort of model is being used to explain the flow attitude of non-Newtonian fluids Chhabra and Richard (2008) which are considered vital and applicable in many industrial producing processes such in the drilling of oil and gas wells, polymer extrusion from dye, glass fibre, paper production and draining of plastics films etc.

A vast scientific analysis of non-Newtonian fluids problems has been carried out by many researchers. This has gained great importance in different fields due to their huge range of engineering and commercial applications. The study of the behaviour of the motion of non-Newtonian fluids is very much more complicated and difficult as compared to that for Newtonian fluids, because of the nonlinear relationship between the stress and the rate of strains. The governing equations that describes the flow of Newtonian fluid is the Navier-Stokes equations, while for the flow of the non-Newtonian fluids there is no single governing equation which describes all of their properties and thus it is difficult to describe these fluids as Newtonian fluids. Therefore, many empirical and semi-empirical non-Newtonian models or constitutive equations have been proposed. Rehan*et al* (2010) considered the steady flow of a fourth grade fluid between two parallel plates. They analysed four types of flows; Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear differential equation describing the velocity field was solved by Optimal Homotopy Asymptotic method (OHAM). They observed that the OHAM is more efficient and flexible than the perturbation and Homotopy analyses method. Islam *et al* (2011) considered the steady flow of a non-Newtonian fluid with slippage between the plate and the fluid. The constitute equations of the fluids are modelled for fourth-grade non-Newtonian fluid with partial slip. The employed homotopy perturbation and optimal homotopy asymptotic methods to solve the non-linear differential equation.

Zaman*et al* (2013) presented solution for unsteady Couette flow problem for the Eyring-Powell model. A strong magnetic field was applied in an ionized gas of low density; the conductivity normal to the magnetic field was decreased by free spiralling of electrons and ions about the magnetic lines of force before suffering collision. Also, Zaman*et al* (2014) analysed the Couette flow problem for an unsteady magnetohydrodynamic (MHD) fouth-grade fluid in the presence of pressure gradient and Hall currents. The arising non-linear problem was solved by the homotopy analysis method (HAM).

Taha*et al* (2015) carried out an analysis to study the time – dependent flow of an incompressible electrically conducting fourth grade fluid over an infinite porous plate. The flow was caused by the motion of the porous plate in its own plane with an impulsive velocity$V(t)$. The governing non – linear problem was solved by invoking the Lie group theoretical approach and numerical technique. In another study, Taza*et al* (2016) studied the unsteady thin film flow of a fourth-grade fluid over a moving and oscillating vertical belt. They employed domain decomposition method (ADM) and optimal homotopy asymptotic method (OHAM) to find the solution of the non-linear differential equations that governed the flow.

Moakher*et al* (2016) studied the incompressible fully developed flow of non-Newtonian fourth grade fluid in a flat channel under externally applied magnetic field. An appropriate analysis was performed by considering the ship condition on the walls. The non-linear equation with robin mixed boundary conditions is solved with collocation (CM) and Least Square Method (LSM). Bhatti and Rashdi (2016) examined the attitude of Williamson nano fluid which was flowing from the stretched surface. Ellahi e*t al* (2018) investigated the impression of nano liquid namely Kerosene-alumina entropy generation also examined with ship influence on Moring. The model of Couette-Poiseuille flow was established by Shehzad*et al* (2018) to analyse aluminium oxide-pvcnano fluid in a chemical. Arifuzzaman*et al* (2019) analysed heat and mass transfer characteristics of naturally corrective hydro-magnetic flows of fourth grade radiative fluid resulting from vertical porous plate. They consider non-linear order chemical reaction and heat generation with thermal diffusion. The complete fundamental equations are transformed into a dimensionless equation by implementing finite difference scheme explicitly. Santhosha*et al* (2017) studied the radiation and chemical reaction effects on MHD free convective heat and mass transfer of a viscoelastic fluid past a porous plate with heat generation/absorption

Idowu and Sani (2019) carried out an analysis for an unsteady magneto hydrodynamic (MHD) flow of a generalized third grade fluid between two parallel plates. The fluid flow is a result of the plate oscillating, moving and pressure gradient. They used He-Laplace method to solve the nonlinear partial differential equations. They found that thermal radiation parameter increases the temperature of the fluid and hence reduces the viscosity of the fluid while the concentration of the fluid reduces as the chemical reaction parameter increases.

Kodi et al. (2023) studied the effects of unsteady natural convection MHD flow with viscous, incompressible, electrically conducting fluid through a porous medium, considering chemical reaction and thermal radiation. The fluid was modeled as a non-Newtonian Jeffrey fluid. The governing equations were solved analytically using a perturbation method, and the results for velocity, concentration, and temperature were analyzed graphically. Numerical values for skin friction, Nusselt number, and Sherwood number were also tabulated. The study's findings have potential applications in solar physics, magnetohydrodynamics, energy generation, and related fields. Mahabaleshwar et al. (2023) worked on the effect ofmagnetohydrodynamic and radiation on axisymmetric flow of non-Newtonian fluid past a porous shrinking/stretching surface,their paper presents an analytical study of the magnetohydrodynamic axisymmetric flow of a Casson fluid over a nonlinear permeable shrinking or stretched surface. The authors investigate the heat transfer characteristics, incorporating thermal radiation effects. Key strengths include the analytical transformation of the governing equations, the use of the Rosseland approximation and incomplete gamma functions, and the graphical analysis of various physical parameters. Dual solutions for velocity, temperature, and skin friction are obtained. However, more discussion on the implications of the dual solutions and validation through numerical/experimental data would further strengthen the work. Overall, the paper contributes to the understanding of magnetohydrodynamic and radiation effects on non-Newtonian fluid dynamics. Joseph et al (2021) investigated the unsteady (MHD) flow of fourth grade fluid in a horizontal parallel plates channel with suction effect. He-Laplace method was used to solve the nonlinear partial differential equation. However, viscous dissipation effect was neglected. In fact, the viscous dissipation effect on heat transfer is significant especially for high velocity flows, because the convection current near the channel is enhanced by viscous dissipation which in return affects the temperature of the fluids, thus causing decrease in fluid density, hence increase the fluid flow. Thus, this research extends the work of Joseph et al (2021). The unsteady magnetohydrodynamic (MHD) flow of fourth grade fluid between stationary plates with viscous dissipation and suction effects has been investigated. The He – Laplace method was employed for solution of the fluid flow model. The effect of various flow parameters such as suction parameter, Eckert number, Prandtl number and Radiation parameter on temperature profile were obtained. Also investigated were the effect of suction parameter, third and fourth grade parameter and Hartman number on velocity profile. Graphs were plotted for each case considered.

MATERIALS AND METHODS

Problem Formulation

We consider the unsteady unidirectional flow an electrically conducting incompressible fourth grade fluid between two horizontal infinite parallel plates channel as shown in Figure 1. The fluid is subjected to a uniform transverse magnetic field. The upper and lower plates are stationary. Introducing the Cartesian coordinate system with the x −axis along the direction of flow and the y −axis perpendicular to it. The flow is generated due to the translation of the upper plate in its own place with impulsive velocity$V(t)$. Since the plates are infinite along the x −axis, all the physical quantities except the pressure depends on y only.

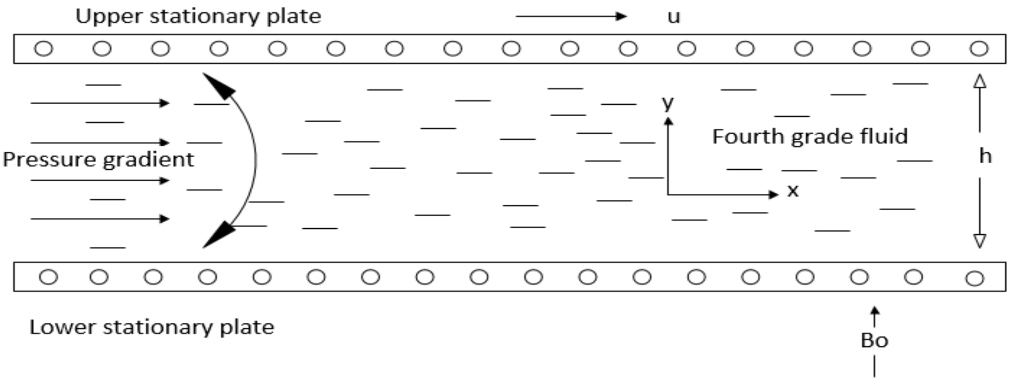

Figure 1: Physical Configuration of the Plane Poiseuille Flow

For the flow model under consideration, we seek a velocity of the form

$$V = [u(y,t), 0, 0] \quad (1)$$

where u denotes the velocity of the fluid in x −direction. It should be noted that Eq.

(1) satisfies the law of conservation of mass for incompressible fluid, i.e.

$$\operatorname{div}(\mathbf{V}) = 0 \quad (2)$$

The unsteady motion of an incompressible non-Newtonian fluid through a porous medium is governed by

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div}\mathbf{T} + \mathbf{R} \quad (3)$$

in which ρ is the fluid density, d/dt the material time derivative, $\boldsymbol{T}$ the Cauchy stress

tensor and $\boldsymbol{R}$ the Darcy's resistance due to porous medium.

The Cauchy stress tensor $\boldsymbol{T}$ for an incompressible fourth grade fluid is

$$\boldsymbol{T} = -p\boldsymbol{I} + \boldsymbol{S_1} + \boldsymbol{S_2} + \boldsymbol{S_3} + \boldsymbol{S_4} \quad (4)$$

where, p is the pressure, $\boldsymbol{I}$ the identity tensor and the extra stress tensors $\boldsymbol{S_1} - \boldsymbol{S_4}$ are

given by

$$S_1 = \mu A_1 \quad (5)$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 \quad (6)$$

$$S_3 = \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1 \quad (7)$$

$$S_4 = \gamma_1 A_4 + \gamma_2(A_3A_1 + A_1A_3) + \gamma_3 A_2^2 + \gamma_4(A_2A_1^2 + A_1^2A_2) + \gamma_5(trA_2)A_2 + \gamma_6(trA_2)A_1^2 + [\gamma_7 trA_3 + \gamma_8 tr(A_2A_1)]A_1 \quad (8)$$

Here, μ is the coefficient of shear viscosity, $\alpha_i (i = 1,2)$, $\beta_j (j = 1,2,3)$ and $\gamma_k (k = 1, \dots, 8)$ are material constants. The A_n are the Rivlin – Ericksen tensors defined by the recursion relation

$$A_1 = L + L^T \quad (9)$$

$$A_n = \frac{d}{dt}A_{n-1} + A_{n-1}L + L^T A_{n-1}, \; n>1 \quad (10)$$

Where, $L = \nabla V$, $\frac{d}{dt}$ is the material time derivative, $V$is the velocity and ∇ is the gradient operator.

Using equations (1), (9) and (10) $A_1 - A_4$ are obtained as

$$A_1 = \begin{pmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (11)$$

$$A_2 = \begin{pmatrix} 0 & \frac{\partial^2 u}{\partial t \partial y} & 0 \\ \frac{\partial^2 u}{\partial t \partial y} & 2\left(\frac{\partial u}{\partial y}\right)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (12)$$

$$A_3 = \begin{pmatrix} 0 & \frac{\partial^3 u}{\partial t^2 \partial y} & 0 \\ \frac{\partial^3 u}{\partial t^2 \partial y} & 2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2 + 2\left(\frac{\partial^3 u}{\partial y^2 \partial t}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (13)$$

$$A_4 = \begin{pmatrix} \frac{\partial^4 u}{\partial t^2 \partial y^2} & \frac{\partial^4 u}{\partial t^3 \partial y} & 0 \\ \frac{\partial^4 u}{\partial t^3 \partial y} & 2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^3 + 2\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial y}\right)^2 + 3\left(\frac{\partial^4 u}{\partial y^2 \partial t^2}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (14)$$

The substitution of Eqs. (11) – (14) into Eqs. (4) - (8) yields the following non-zero components of the Cauchy stress tensor

$$T_{xx} = -p + \alpha_2\left(\frac{\partial u}{\partial y}\right)^2 + 2\beta_2 \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t \partial y} + 2\gamma_2 \frac{\partial u}{\partial y}\frac{\partial^3 u}{\partial y \partial t^2} + \gamma_3\left(\frac{\partial^2 u}{\partial t \partial y}\right)^2 + 2\gamma_6\left(\frac{\partial u}{\partial y}\right)^4 \quad (15)$$

$$T_{xy} = T_{yx} = \mu\frac{\partial u}{\partial y} + \alpha_1\frac{\partial^2 u}{\partial y \partial t} + \beta_1\frac{\partial^3 u}{\partial y \partial t^2} + 2(\beta_2 + \beta_3)\left(\frac{\partial u}{\partial y}\right)^3 + \gamma_1\frac{\partial^4 u}{\partial y \partial t^3} + 2(3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_7 + \gamma_8)\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial y \partial t} \quad (16)$$

$$T_{yy} = -p + (2\alpha_1 + \alpha_2)\left(\frac{\partial u}{\partial y}\right)^2 + (6\beta_1 + \beta_2)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t \partial y} + (4\gamma_4 + 4\gamma_5 + 2\gamma_6)\left(\frac{\partial u}{\partial y}\right)^4 + \gamma_1\left[6\left(\frac{\partial^2 u}{\partial y \partial t}\right)^2 + 8\frac{\partial u}{\partial y}\frac{\partial^3 u}{\partial y \partial t^2}\right] + 2\gamma_2\frac{\partial u}{\partial y}\frac{\partial^3 u}{\partial y \partial t^2} + \gamma_3\left[\left(\frac{\partial^2 u}{\partial y \partial t}\right)^2 + 4\left(\frac{\partial u}{\partial y}\right)^4\right] \quad (17)$$

To incorporate the effects of the pores on the velocity field, we have made use of generalized Darcy's law consistent with the thermodynamics stability of the flow. This law relates the pressure drop induced by the fractional drag and velocity and ignores the boundary effects of the flow.

Since the pressure gradient can be interpreted as a measure of the flow resistance in the bulk of the porous medium and Darcy's resistance R is the measure of the resistance to the flow in the porous media. According to Darcy's law, we have

$$\boldsymbol{R} = \boldsymbol{\nabla p} = -\frac{\phi}{k}\boldsymbol{V} \quad (18)$$

where, ϕ is the porosity and k the permeability of the porous medium. For different non-Newtonian fluid flows the apparent viscosity is different. By making use of Eq. (16), the apparent viscosity for unsteady unidirectional flow of a fourth-grade fluid over a rigid plate is calculated as

$$\text{Apparent Viscosity} = \frac{\text{Shear Stress}}{\text{Shear Rate}} = \mu + \alpha_1\frac{\partial}{\partial t} + \beta_1\frac{\partial^2}{\partial t^2} + 2(\beta_2 + \beta_3)\left(\frac{\partial u}{\partial y}\right)^2 + \gamma_1\frac{\partial^3}{\partial t^3} + 2(3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_7 + \gamma_8)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y \partial t} \quad (19)$$

With the use of Eq. (18) into Eq. (17), the x −component of $\boldsymbol{R}$ for the unidirectional flow over a rigid plate is given by

$$\mathbf{R_x} = -\frac{\phi}{k}\left[\mu + \alpha_1\frac{\partial}{\partial t} + \beta_1\frac{\partial^2}{\partial t^2} + 2(\beta_2 + \beta_3)\left(\frac{\partial u}{\partial y}\right)^2 + \gamma_1\frac{\partial^3}{\partial t^3} + 2(3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_7 + \gamma_8)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y \partial t}\right]u \quad (20)$$

Finally, by making use of Eq. (20) and Eqs. (15) - (17) into Eq. (3); one deduces the following governing equation

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1 \nu}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\beta_1 \nu^2}{\rho}\frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \frac{\gamma_1 \nu^3}{\rho}\frac{\partial^5 u}{\partial y^2 \partial t^3} + \frac{2\nu(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_7+\gamma_8)}{\rho C_p}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2 \partial t}\right] - \frac{\sigma B_0^2}{\rho C_p}u - \frac{\nu}{k}u \quad (21)$$

The of conservation of energy equation is given as

$$\rho C_p \frac{DT}{Dt} = k\nabla^2 T + \phi - \frac{\partial q_r}{\partial y} \quad (22)$$

Where, c_p is the specific heat at constant pressure, ρ is the density,k is the thermal conductivity, ϕ is the dissipation function, q_r is the radiative heat flux.

The dissipation ϕ and radiative heat flux $\frac{\partial q_r}{\partial y}$ in Eq. (22) are defined as follows;

$$\phi = \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (23)$$

$$\frac{\partial q_r}{\partial y} = 4\alpha^2(T_w - T) \quad (24)$$

Thus, the energy equation from Eq. (3.8) for unidirectional flow becomes

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial t^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y} \quad (25)$$

Eqs. (21) and (25) are the equations that governs the flow which are respectively the momentum and energy equations.

Now, as elucidated by Joseph *et al* (2021), the initial and boundary conditions are

$$\left.\begin{array}{lll} u(y,t)=h+h\cos(\omega y), \quad T(y,t)=h+h\cos(\omega y) & \text{at} \quad t=0 & \text{for} \quad 0\le y\le h, \\ u(y,t)=U, \quad T(y,t)=T_w & \text{at} \quad y=h & \text{for} \quad t\ge 0, \\ u(y,t)\to\infty, \quad T(y,t)\to\infty & \text{as} \quad y\to\infty & \text{for} \quad t>0. \end{array}\right\} \tag{26}$$

In order to transform equations (21), (25) and (26) into dimensional form, we use the following dimensionless parameters

$$\bar{u}=\frac{u}{U_0}, \theta=\frac{T-T_0}{T_w-T_0}, \bar{t}=\frac{t}{\uparrow\upsilon U_0^2}, \bar{y}=\frac{yU_0}{\upsilon}, P_r=\frac{\rho C_p}{k\upsilon}, \ N=\frac{4\alpha^2\upsilon}{\rho C_p U_0^2}$$

$$h=\frac{U_0}{\upsilon}, S=\frac{\upsilon_0}{U_0}, \bar{x}=\frac{x}{h}, EC=\frac{U_0^2}{C_p(T_w-T_0)}, Da=\frac{kU_0^2}{\upsilon^2} \tag{27}$$

Substituting equation (27) into equations (21), (25) & (26) and by dropping the bars, we have the following transformed governing equations

$$\frac{\partial u}{\partial t}-\mathrm{S}\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u \text{ where}$$

$$l_1=M^2+\frac{1}{Da} \tag{28}$$

$$\frac{\partial\theta}{\partial t}-s\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial^2 u}{\partial y^2}\right)^2+N\theta \tag{29}$$

And the initial and boundary conditions become

$$\left.\begin{array}{lll} u(y,t)=1+\cos(\omega y), \quad T(y,t)=1+\cos(\omega y) & \text{at} \quad t=0 & \text{for} \quad 0\le y\le h, \\ u(y,t)=U, \quad T(y,t)=T_w & \text{at} \quad y=h & \text{for} \quad t\ge 0, \\ u(y,t)\to\infty, \quad T(y,t)\to\infty & \text{as} \quad y\to\infty & \text{for} \quad t>0. \end{array}\right\} \tag{30}$$

Solution of the Problem

In this section we employed the He – Laplace Scheme to solve equations (28) and (29) subject to the initial and boundary conditions (30)

Applying the Laplace transform on both sides of equation (28) gives;

$$L\left\{\frac{\partial u}{\partial t}\right\}-L\left\{\mathrm{S}\frac{\partial u}{\partial y}\right\}=L\left\{-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\} \tag{31}$$

But,

$$L\left\{\frac{\partial u}{\partial t}\right\}=sL\{u(y,t)\}-u(y,0) \tag{32}$$

We have,

$$sL\{u(y,t)\}-u(y,0)-L\left\{\mathrm{S}\frac{\partial u}{\partial y}\right\}=L\left\{-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\right.$$
$$\left.\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\} \tag{33}$$

Hence,

$$L\{u(y,t)\}=\frac{u(y,0)}{s}+\frac{1}{s}L\left\{-\frac{\partial p}{\partial x}+\mathrm{S}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\right.\right.$$
$$\left.\left.\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\} \tag{34}$$

Applying the initial condition and rearranging, we have,

$$L\{u(y,t)\}=\frac{1+\cos(\omega y)}{s}+\frac{\lambda}{s}+\frac{1}{s}L\left\{\mathrm{S}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\right.\right.$$
$$\left.\left.\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\} \tag{35}$$

Taking the inverse Laplace transform of both sides of equation (35), we have;

$$L^{-1}L\{u(y,t)\}=L^{-1}\left\{\frac{1+\cos(\omega y)}{s}+\frac{\lambda}{s}\right\}+L^{-1}\left\{\frac{1}{s}\left\{\mathrm{S}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\right.\right.$$
$$\left.\left.\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\}\right\} \tag{36}$$

$$u(y,t)=1+\lambda+\cos(\omega y)+L^{-1}\left\{\frac{1}{s}\left\{\mathrm{S}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\right.\right.\right.$$
$$\left.\left.\left.\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\}\right\} \tag{37}$$

Applying the Homotopy perturbation method to equation (37), gives,

$$\sum_{n=0}^{\infty}P^n u_n(y,t)=1+\lambda+\cos(\omega y)+P\left(L^{-1}\left\{\frac{1}{s}\left\{\mathrm{S}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3}+\right.\right.\right.$$
$$\left.\left.\left.\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right]-l_1 u\right\}\right\}\right) \tag{38}$$

Or,

$$\sum_{n=0}^{\infty} P^n u_n(y,t) = 1+\lambda+\cos(\omega y) + P\left(L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u}{\partial y^2\partial t^2}+\beta_b H_a(u_n)+\gamma_1\frac{\partial^5 u}{\partial y^2\partial t^3}+\gamma_b[2H_b(u_n)+H_c(u_n)]-l_1 u\right\}\right\}\right) \quad (39)$$

Where, $H_a(u_n), H_b(u_n)$ and $H_c(u_n)$ are the He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2, \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}$ and $\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}$ respectively.

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2$ are;

$$\begin{cases} H_0(u) = (u_0')^2 \\ H_1(u) = 2u_0'u_1' \\ H_2(u) = 2u_0'u_2' + (u_1')^2 \\ H_3(u) = 2u_1'u_2' \\ \vdots \end{cases} \quad (40)$$

The He's polynomials for $\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}$ are;

$$\begin{cases} H_0(u) = u_0'''u''_{0t} \\ H_1(u) = u_0'''u''_{1t} + u_1'''u''_{1t} \\ H_2(u) = u_0'''u''_{2t} + u_1'''u''_{1t} + u_2'''u''_{0t} \\ H_3(u) = u_1'''u''_{2t} + u_2'''u''_{1t} \\ \vdots \end{cases} \quad (41)$$

While, the He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}$ are;

$$\begin{cases} H_0(u) = (u_0')^2(u_0''u_{0t}') \\ H_1(u) = (u_0')^2(u_0''u_{1t}') + (u_0')^2(u_1''u_{0t}') + 2u_0'u_1'(u_0''u_{0t}') \\ H_2(u) = (u_0')^2(u_0''u_{2t}') + (u_0')^2(u_1''u_{1t}') + (u_0')^2(u_2''u_{0t}') \\ \quad +2u_0'u_1'(u_0''u_{1t}') + 2u_0'u_1'(u_1''u_{0t}') + 2u_0'u_2'(u_0''u_{0t}') \\ \vdots \end{cases} \quad (42)$$

Now, comparing the like powers of "P" in equation (39) and equating their coefficients gives

$$P^0; u_0(y,t) = 1+\lambda+\cos(\omega y) \quad (43)$$

$$P^1; u_1(y,t) = L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u_0}{\partial y}+\frac{\partial^2 u_0}{\partial y^2}+\alpha\frac{\partial^3 u_0}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u_0}{\partial y^2\partial t^2}+\beta_b H_a(u_0)+\gamma_a\frac{\partial^5 u_0}{\partial y^2\partial t^3}+\gamma_b[2H_b(u_0)+H_c(u_0)]-l_1u_0\right\}\right\}$$

$$u_1(y,t) = L^{-1}\left\{\frac{1}{s}L\{[-\omega^2\cos(\omega y)]+[S(-\omega\sin(\omega y))]+[\beta_a\omega^2\sin^2(\omega y)]-[l_1(1+\lambda+\cos(\omega y))]\}\right\}$$

$$u_1(y,t) = L^{-1}\left\{\frac{1}{s^2}\{[-\omega^2\cos(\omega y)]+[S(-\omega\sin(\omega y))]+[\beta_a\omega^2\sin^2(\omega y)]-[l_1(1+\lambda+\cos(\omega y))]\}\right\}$$

$$u_1(y,t) = (\beta_a\omega^2\sin^2(\omega y)-\omega^2\cos(\omega y)-S\omega\sin(\omega y)+l_1(1+\lambda+\cos(\omega y)))t \quad (44)$$

$$P^2; u_2(y,t) = L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u_1}{\partial y}+\frac{\partial^2 u_1}{\partial y^2}+\alpha\frac{\partial^3 u_1}{\partial y^2\partial t}+\beta_a\frac{\partial^4 u_0}{\partial y^2\partial t^2}+\beta_b H_a(u_1)+\gamma_a\frac{\partial^5 u_1}{\partial y^2\partial t^3}+\gamma_b[2H_b(u_1)+H_c(u_1)]-l_1u_1\right\}\right\}$$

$$u_2(y,t) = L^{-1}\left\{\frac{1}{s}L\{(2\beta_b\omega^4\cos(2\omega y)+\omega^4\cos(\omega y)+\omega^3\sin(\omega y)+l_1\omega^2\cos(\omega y))t\} + SL\{(2\beta_b\omega^3\cos(\omega y)\sin(\omega y) - \omega^2\cos(\omega y)+\sin(\omega y)+l_1\omega\sin(\omega y))t\} + L\{\left(-4\beta_b^2\omega^4\sin^2(\omega y)\cos(\omega y) - 4\beta_b\omega^4\sin^2(\omega y) + 2\beta_b\omega^3\cos(\omega y)\sin(\omega y) - \beta_b l_1\omega^2\sin^2(\omega y)\right)t\} + \gamma_b L\left\{\left(2\beta_b\omega^5\cos(\omega y)\sin^3(\omega y) - 2\omega^5\cos(\omega y)\sin(\omega y) + 2S\omega^4\sin^2(\omega y) - 2l_1\omega^3\sin(\omega y)(1+\lambda+\cos(\omega y)) - \beta_b^2\omega^6\sin^4(\omega y)\cos(\omega y) + \omega^6\sin^2(\omega y)\cos^2(\omega y) + S\omega^5\sin^3(\omega y)\cos(\omega y) + l_1\omega^4\sin^2(\omega y)\cos(\omega y)(1+\lambda+\cos(\omega y))\right)t\right\} - l_1L\{(\beta_a\omega^2\sin^2(\omega y) - S\omega\sin(\omega y) + l_1(1+\lambda+\cos(\omega y)))t\}\right\}$$

$$u_2(y,t) = \Big(2\beta_b\omega^4\cos(2\omega y)+\omega^4\cos(\omega y)+\omega^3\sin(\omega y)+l_1\omega^2\cos(\omega y) - 4\beta_b^2\omega^4\sin^2(\omega y)\cos(\omega y) - 4\beta_b\omega^4\sin^2(\omega y) + 2\beta_b\omega^3\cos(\omega y)\sin(\omega y) - \beta_b l_1\omega^2\sin^2(\omega y) + S(2\beta_b\omega^3\cos(\omega y)\sin(\omega y) - \omega^2\cos(\omega y)+\sin(\omega y)+l_1\omega\sin(\omega y)) + \gamma_b\Big(2\beta_b\omega^5\cos(\omega y)\sin^3(\omega y) - 2\omega^5\cos(\omega y)\sin(\omega y) + 2S\omega^4\sin^2(\omega y) - 2l_1\omega^3\sin(\omega y)(1+\lambda+\cos(\omega y)) - \beta_b^2\omega^6\sin^4(\omega y)\cos(\omega y) + \omega^6\sin^2(\omega y)\cos^2(\omega y) + S\omega^5\sin^3(\omega y)\cos(\omega y) + l_1\omega^4\sin^2(\omega y)\cos(\omega y)(1+\lambda+\cos(\omega y))\Big) - l_1(\beta_a\omega^2\sin^2(\omega y) - S\omega\sin(\omega y) + l_1(1+\lambda+\cos(\omega y)))\Big)\frac{t^2}{2!} \quad (45)$$

Therefore, the solution to the velocity profile is

$$u(y,t) = u_0(y,t) + u_1(y,t) + u_2(y,t) + \cdots$$

$u(y,t) = 1 + \lambda + \cos(\omega y) + (\beta_a \omega^2 \sin^2(\omega y) - \omega^2 \cos(\omega y) - S\omega \sin(\omega y) + l_1(1 + \lambda + \cos(\omega y)))t + \Big(2\beta_b \omega^4 \cos(2\omega y) + \omega^4 \cos(\omega y) + \omega^3 \sin(\omega y) + l_1 \omega^2 \cos(\omega y) - 4\beta_b^2 \omega^4 \sin^2(\omega y)\cos(\omega y) - 4\beta_b \omega^4 \sin^2(\omega y) + 2\beta_b \omega^3 \cos(\omega y)\sin(\omega y) - \beta_b l_1 \omega^2 \sin^2(\omega y) + S(2\beta_b \omega^3 \cos(\omega y)\sin(\omega y) - \omega^2 \cos(\omega y) + \sin(\omega y) + l_1 \omega \sin(\omega y)) + \gamma_b \Big(2\beta_b \omega^5 \cos(\omega y)\sin^3(\omega y) - 2\omega^5 \cos(\omega y)\sin(\omega y) + 2S\omega^4 \sin^2(\omega y) - 2l_1 \omega^3 \sin(\omega y)(1 + \lambda + \cos(\omega y)) - \beta_b^2 \omega^6 \sin^4(\omega y)\cos(\omega y) + \omega^6 \sin^2(\omega y)\cos^2(\omega y) + S\omega^5 \sin^3(\omega y)\cos(\omega y) + l_1 \omega^4 \sin^2(\omega y)\cos(\omega y)(1 + \lambda + \cos(\omega y))\Big) - l_1(\beta_a \omega^2 \sin^2(\omega y) - S\omega \sin(\omega y) + l_1(1 + \lambda + \cos(\omega y)))\Big)\frac{t^2}{2!}$ (46)

Next, we consider the energy equation (29), which is rearranged to give;

$$\frac{\partial\theta}{\partial t} - s\frac{\partial\theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} + Ec\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + N\theta \quad (47a)$$

Now applying Laplace transform on equation (), we get,

$$L\left\{\frac{\partial\theta}{\partial t}\right\} - L\left\{S\frac{\partial\theta}{\partial y}\right\} = L\left\{\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}\right\} + L\left\{Ec\left(\frac{\partial^2 u}{\partial y^2}\right)^2\right\} + L\{N\theta\} \quad (47b)$$

Applying the initial condition and dividing through by s and rearranging we obtain;

$$L\{\theta(y,t)\} = \frac{1+\cos(\omega y)}{s} + \frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\} + L\left\{S\frac{\partial\theta}{\partial y}\right\} + L\left\{Ec\left(\frac{\partial^2 u}{\partial y^2}\right)^2\right\} + L\{N\theta\}\right\} \quad (48)$$

Taking the inverse Laplace transform of both sides of equation (48) gives,

$$\theta(y,t) = 1 + \cos(\omega y) + L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\} + L\left\{S\frac{\partial\theta}{\partial y}\right\} + L\left\{Ec\left(\frac{\partial^2 u}{\partial y^2}\right)^2\right\} + L\{N\theta\}\right\}\right] \quad (49)$$

Applying the Homotopy perturbation technique on equation (49), yields

$$\sum_{n=0}^{\infty} P^n\theta_n(y,t) = 1 + \cos(\omega y) + P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta}{\partial y}\right\} + EcL\left\{\left(\frac{\partial u}{\partial y}\right)^2\right\} + L\{N\theta\}\right\}\right\}\right] \quad (50)$$

Or, $$\sum_{n=0}^{\infty} P^n\theta_n(y,t) = 1 + \cos(\omega y) + P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta}{\partial y}\right\} + EcL\{H_n(u)\} + L\{N\theta\}\right\}\right\}\right] \quad (51)$$

Where, $H_a(u_n)$, for the nonlinear term $\left(\frac{\partial u}{\partial y}\right)^2$

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2$ are;

$$\begin{cases} H_0(u) = (u_0')^2 \\ H_1(u) = 2u_0'u_1' \\ H_2(u) = 2u_0'u_2' + (u_1')^2 \\ H_3(u) = 2u_1'u_2' \\ \vdots \end{cases} \quad (52)$$

Comparing the coefficients of the like powers of $'P'$in equation (51), the following approximations are obtained;

$$P^0: \theta_0(y,t) = 1 + \cos(\omega y) \quad (53)$$

$$P^1: \theta_1(y,t) = L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_0}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta_0}{\partial y}\right\} + EcL\{H_0(u)\} + L\{N\theta_0\}\right\}\right\}$$

$$\theta_1(y,t) = L^{-1}\left\{\frac{1}{s}\left\{L\left[\frac{-1}{Pr}\omega^2\cos(\omega y)\right] + L[-S\omega\sin(\omega y)] + L[Ec\omega^2\sin^2(\omega y)] + L[N(1 + \cos(\omega y))]\right\}\right\} \quad (54)$$

Or

$$\theta_1(y,t) = \left(Ec\omega^2\sin^2(\omega y) - \frac{1}{Pr}\omega^2\cos(\omega y) - S\omega\sin(\omega y) + N + N\cos(\omega y)\right)t \quad (55)$$

$$P^2: \theta_2(y,t) = L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_1}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta_1}{\partial y}\right\} + EcL\{H_1(u)\} + L\{N\theta_1\}\right\}\right\} \quad (56)$$

$\theta_2(y,t) = \Big(\frac{2Ec\omega^4}{P_r}\cos(2\omega y) - \frac{2\omega^4}{P_r^2}\cos(\omega y) + \frac{S\omega^3}{Pr}\sin(\omega y) - \frac{N\omega^2\cos(\omega y)}{Pr} + 2EcS\omega^3\sin(\omega y)\cos(\omega y) + \frac{S}{Pr}\omega^3\sin(\omega y) - S^2\omega^2\cos(\omega y) - N\omega\sin(\omega y) - 4\beta_b^3\omega^4\cos(\omega y)\sin^2(\omega y) - 2\beta_b^2\omega^4\sin^2(\omega y) + 2\beta_b^2\omega^3\sin(\omega y)\cos(\omega y) - 2\beta_b^2 l_1\omega^2\sin^2(\omega y) - Ecl_1\omega^2\sin^2(\omega y) - \frac{l_1}{Pr}\omega^2\cos(\omega y) - l_1S\omega\sin(\omega y) + l_1N + l_1N\cos(\omega y)\Big)\frac{t^2}{2!}$ (57)

Therefore, the solution to the temperature profile is

$\theta(y,t) = \theta_0(y,t) + \theta_1(y,t) + \theta_2(y,t) + \theta_3(y,t)\cdots$

$\theta(y,t) = 1 + \cos(\omega y) + \Big(Ec\omega^2\sin^2(\omega y) - \frac{1}{Pr}\omega^2\cos(\omega y) - S\omega\sin(\omega y) + N + N\cos(\omega y)\Big)t + \Big(\frac{2Ec\omega^4}{P_r}\cos(2\omega y) - \frac{2\omega^4}{P_r^2}\cos(\omega y) + \frac{S\omega^3}{Pr}\sin(\omega y) - \frac{N\omega^2\cos(\omega y)}{Pr} + 2EcS\omega^3\sin(\omega y)\cos(\omega y) + \frac{S}{Pr}\omega^3\sin(\omega y) - S^2\omega^2\cos(\omega y) - N\omega\sin(\omega y) - 4\beta_b^3\omega^4\cos(\omega y)\sin^2(\omega y) - 2\beta_b^2\omega^4\sin^2(\omega y) + 2\beta_b^2\omega^3\sin(\omega y)\cos(\omega y) - 2\beta_b^2 l_1\omega^2\sin^2(\omega y) - Ecl_1\omega^2\sin^2(\omega y) - \frac{l_1}{Pr}\omega^2\cos(\omega y) - l_1S\omega\sin(\omega y) + l_1N + l_1N\cos(\omega y)\Big)\frac{t^2}{2!}$ (58)

RESULTS AND DISCUSSION

The unsteady magnetohydrodynamics (MHD) flow of fourth grade fluid between stationary plates with suction and viscous dissipation effects has been investigated. The impact of pertinent flow parameters such as suction parameter, Hartmann number, Eckert number, third and fourth grade parameters, Prandtl number, radiation parameter are plotted graphically on different flow fields. The default values for the pertinent flow parameters are taken as Joseph et al (2021)
$\lambda = 0.30, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05, S = 0.10$
$M = 0.30, \gamma_a = 0.05, \gamma_b = 0.05, P_r = 0.71, Da = 1.00, t = 0.02, Ec = 0.005$

The impact of the suction parameter S on velocity and temperature profiles is illustrated in Figures 2 and 3 respectively. It is clearly seen that both the velocity and temperature fields diminish with an increase of suction parameter S. This is due to the porosity of the channel.

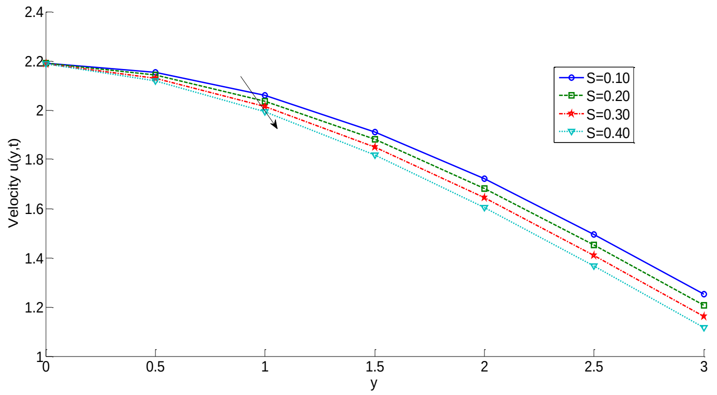

Figure 2: Effect of Suction Parameter S on Velocity Profile $u(y,t)$

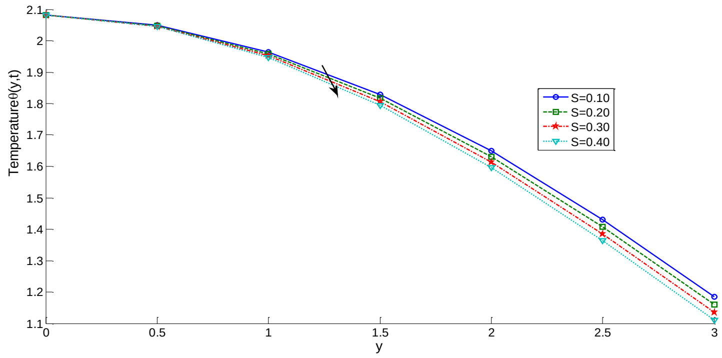

Figure 3: Effect of Suction Parameter S on Temperature profile $\theta(y,t)$

The impression of third – grade parameter (β_a and β_b) and fourth -grade parameter (γ_a and γ_b) is depicted in Figures 4 and 5 respectively. It is observed that the velocity field increases with increase in both third and fourth grade parameters. This also clearly shown that the fluid is purely fourth grade.

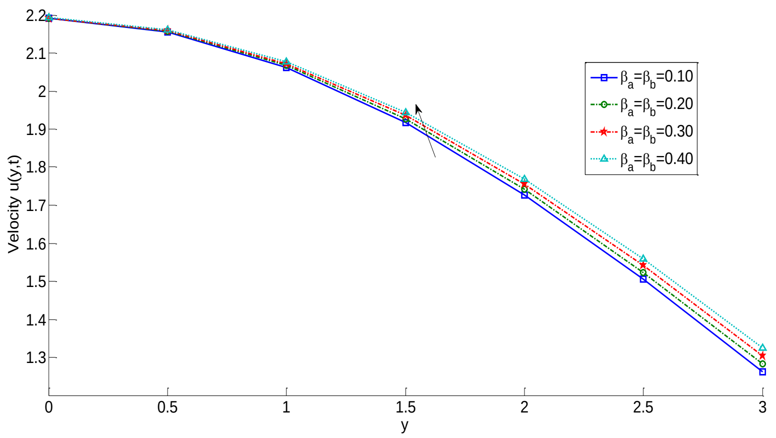

Figure 4: Effect of Third Grade Parameters β_a and β_b on Velocity Profile $u(y,t)$

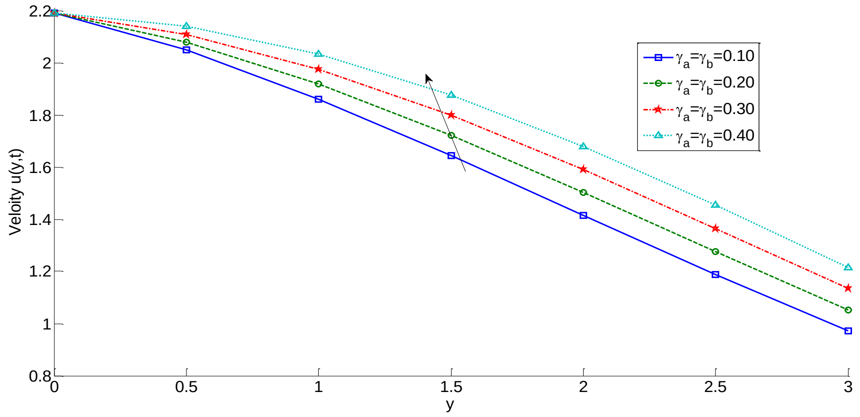

Figure 5: Effect of Fourth Grade Parameters γ_a and γ_b on Velocity Profile $u(y,t)$

Figure 6 illustrates the effect of Hartmann number M on velocity profile. The velocity field decreases with the increment of Hartmann number. The role of Hartmann number which is the magnetic field is to suppress turbulence. Physically, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming viscous elastic solid. It is of great interest that yields stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of electromagnet which give rise to many possible control – based applications, including MHD power generation, electromagnetic casting of metals, MHD propulsion etc.

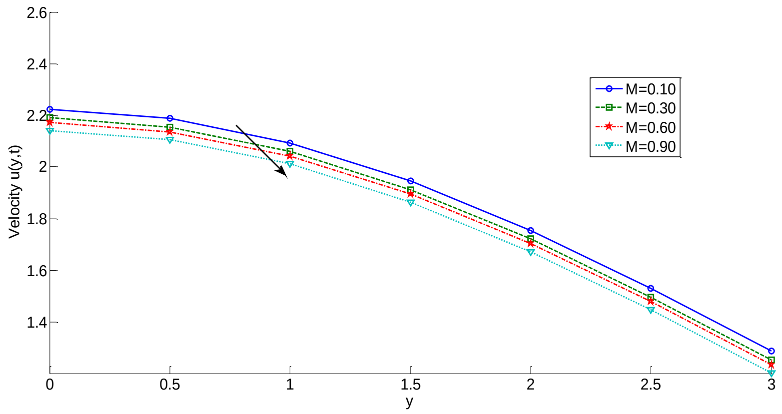

Figure 6: Effect of Hartmann Number M on Velocity Profile $u(y,t)$

The Eckert number Ec which is the dimensionless number that expresses the relationship between a flow kinetic energy and enthalpy. It is also used to characterise dissipation. The effect of Eckert number is demonstrated in Figure 7. It is clearly seen that the temperature profile increases with increase in Eckert number.

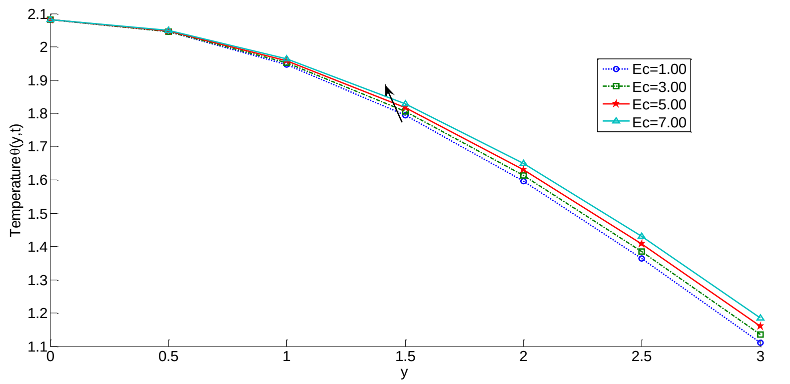

Figure 7: Effect of Eckert Number Ec on Temperature Profile $\theta(y,t)$

Figure 8 shows the impression of Prandtl number P_r on temperature profile. The parameter (P_r) is the proportion of kinematic viscosity and thermal diffusivity which changes physically with temperature. Prandtl number is used to determine whether heat transport occurs with either conduction or convection process. Since, Prandtl number is inversely proportional to the thermal diffusivity so that increasing P_r led to the decrease in temperature profile.

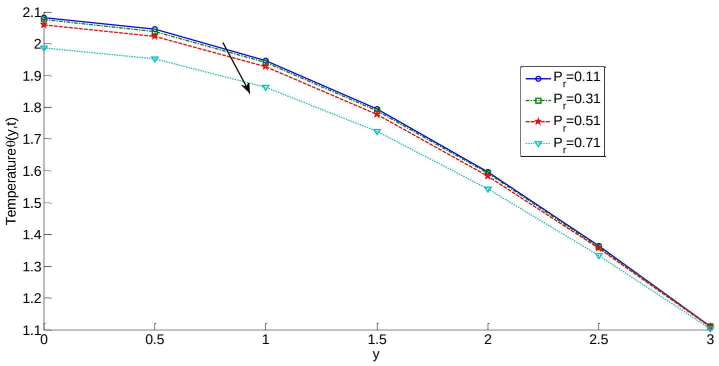

Figure 8: Effect of Prandtl Number P_r on Temperature Profile $\theta(y,t)$

The effect of radiation parameter N on temperature profile is depicted in Figure 9. It can be seen that the temperature profile increases with the increment of radiation parameter. Thermal radiation is known as electromagnetic radiation or the conversion of thermal energy which generates the thermal motion of particles in matter. Thermal radiation could be attributed due to thermal excitation.

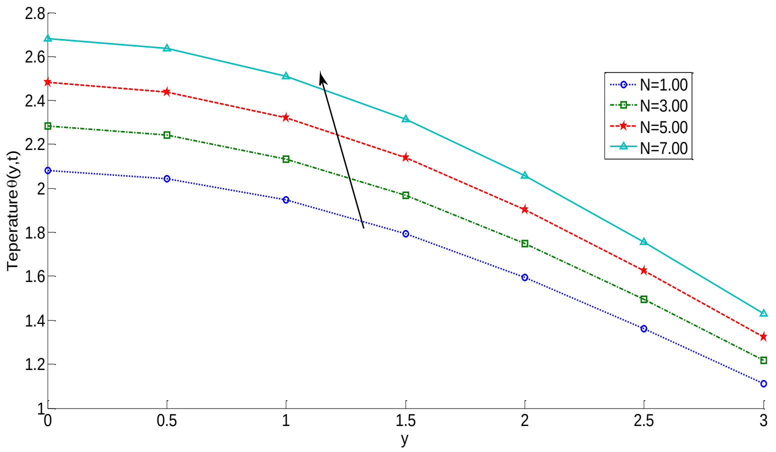

Figure 9: Effect of Radiation Parameter Non Temperature Profile $\theta(y,t)$

CONCLUSION

The unsteady magnetohydrodynamics (MHD) flow of fourth grade fluid between stationary plates with suction and viscous dissipation effects was investigated. The governing equations for the flow considered were obtained from the basic equations of fluid motion. The resulting partial differential equation were solved using the He-Laplace method, which is a combination of homotophy perturbation and Laplace method, to obtain the velocity and temperature profile respectively.

The findings of the study revealed that:

i. Velocity and temperature profile diminish due to the increment of suction parameter.
ii. Temperature profile rise due to the increment of Eckert number.
iii. Velocity goes up when the third and fourth grade parameters get to rise.
iv. Velocity field diminish due to the increment of magnetic parameter.
v. Increase in Prandtl number decline the temperature distribution.
vi. Temperature distribution rises due to the increment of thermal radiation parameter.

Nomenclature	
Bo	External magnetic field
T	Temperature of the fluid
qr	Radiative heat flux
u	Fluid velocity
S	Suction parameter

Nomenclature	
Nu	Nusselt number
Ha	Hartmann number
Pr	Prandtl number
Tw	Temperature at the surface
$T\infty$	Ambient temperature as $y \to \infty$

Greek Symbols	
μ	Coefficient of shear viscosity
α	Second grade parameter
β_a, β_b	Third grade parameters
γ_a, γ_b	Fourth grade parameters
β	Thermal expansion coefficient
δ	Thermal radiation parameter
σ	Stefan – Boltzmann constant
ρ	Density of the fluid

REFERENCES

Arifuzzaman, S. M., Shakhaoth, K. Md., Al-Mamum, A., Reza- E-Rabbi, S. K., Biswas, P. and Karim, I. (2019). Hydrodynamic Stability and Heat and Mass Transfer Flow Analysis of MHD Radiative Fourth-Grade Fluid Through porous Plate with Chemical Reaction. Journal of King Saud University, 31(4), 1388 – 1398.

Aziz, T. & Mahomed, F. M. (2013). Reductions and Solutions for the Unsteady Flow of Fourth-Grade Fluid on a Porous Plate. Appli. Maths. Comput., 219, 9187 – 9195.

Bastian, E. R. (2017). Microfluidics: Modelling, Mechanics and Mathematics, Elsevier Imprint.

Bhatti, M. M. & Rashidi, M. M. (2016). Effects of Thermo-diffusion and Thermal Radiation on Williamson Nano Fluid over a Porous Shrinking/Stretching Sheet. J. Mol. Liq., 221, 567 – 573.

Cengel Y. A. (2003). Heat and mass transfer: a practical approach, 3rd edn. McGraw-Hill, Boston.

Chhabra, R. P. & Richard, S. F. (2008). Non – Newtonian Flow and Applied Rheology. 2ndedtion, Butterworth – Heinemann.

Cole, B.D. & Noll, W. (1960). An Approximation Theorem for Functionals with Applications in Continuum Mechanics. Arch. Ration. Mech. Anal., 6, 355 – 370.

Dunn, J. E. & Rajagopal, K. R. (1995) Fluids of Differential Type: Critical Review and Thermodynamic Analysis. International Journal of Engineering Science, 33(5), 689 – 729.

Ellahi, R., Zeeshan, A., Shehzad, N. & Alauri, S. Z. (2018). Structural Impact of Kerosene – Al_2O_3 Nano Liquid on MHD Poiseuille Flow with Variable Thermal Conductivity: Application of Cooling Process. J. Mol. Liq., 264, 607 – 615.

Frank, M. W. (1990). Fluid Mechanics, Fourth Edition McGraw HillFourier (1822). TheorieAnalytique de la Chaleurin

Ghorbani, A. (2009). Beyond Adomian Polynomials: He's Polynomials. Chaos Solutions and Fractions, 39(3), 1486 – 1492.

Ghorbani, A. & Saberi-Nadjafi, J. (2007). He's Homotopy Perturbation Method for Calculating Adomian Polynomials. International Journal of Nonlinear Science and Numerical Simulation, 8, 229 – 232.

Hayat, T., Wang, Y. & Hutter, K. (2002). Flow of a Fourth-Grade Fluid. Math.Models Methods Appl. Sci., 12, 797 – 811.

Hayat, T., Kara, A. H., & Momoniat, E. (2005). The Unsteady Flow of a Fourth-Grade Fluid Past a Porous Plate. Mathematical and Computer Modeling, 41, 1347 – 1353.

Hradesh, K. M. & Atulya, K. N. (2012). He-Laplace Method for Linear and Nonlinear Partial Differential Equations. Hindawi Publishing Corparation Journal of Applied Mathematics, 20, 1 – 16.

Huigol, R. (1975). Continuum Mechanics of Viscoelastic Liquid, Hindusthan.

Idowu, A.S & Sani, U. (2019). Thermal Radiation and Chemical Reaction Effects on Unsteady MagnetohydrodynamicThird Grade Fluid Flow Between Stationary and Oscillating Plates. International Journal of Applied Mechanics and Engineering, 24(2), 269 – 293.

Islam, S., Bano, Z., Siddique, I. & Siddiqui, A. M. (2011). The Optimal Solutions for the Flow of a Fourth-Grade Fluid with Partial Slip. Computer and Mathematics with Application, 6, 1507 – 1516.

Joseph, K. M., Ayankop, A. E. & Muhammed, S. U. (2021). Unsteady (MHD) Flow of Fourth Grade Fluid in a Horizontal Parallel Plates Channel with Suction Effect. International journal of applied mechanics and engineering, 26, 77 – 98.

Khan, Z., Khan, I., Ullah, M. & Tlili, I. (2018). Effect of Thermal Radiation and Chemical Reaction on non – Newtonian Fluid through a Vertically Stretching Porous Plate with Uniform Suction. Results in Physics, 9, 1086 – 1095.

Kodi, R., Vaddemani, R. R., Khan, M. I., Abdullaev, S. S., Habibullah, Boudjemline, A., Boujelbene, M., & Bouazzi, Y. (2023). Unsteady magneto-hydro-dynamics flow of Jeffrey fluid through porous media with thermal radiation, Hall current and Soret effects. Journal of Magnetism and Magnetic Materials, 582, 171033. https://doi.org/10.1016/j.jmmm.2023.171033

Mahabaleshwar, U. S., Maranna, T., Pérez, L. M., & Nayakar, S. N. R. (2023). An effect of Magnetohydrodynamic and Radiation on Axisymmetric Flow of non-Newtonian Fluid Past a Porous Shrinking/Stretching Surface. Journal of Magnetism and Magnetic Materials, 571, 170538. https://doi.org/10.1016/j.jmmm.2023.170538

Moakher, G. R., Abbasi, M. & Khaki, M. (2016). Fully Developed Flow of Fourth-Grade Fluid through a Channel with Slip Condition in the Presence of a Magnetic Field. Journal of Applied Fluid Mechanics, 9(5), 2239 – 2245.

René, M. & Sergei, M. S. (2007). Julius Hartmann and His Followers: A Review on the Properties of the Hartmann Layer. Magnetohydrodynamics: Historical Evolution and Trends. Fluid Mechanics and Its Application, 80, 155 – 156.

Rehan, A.S., Islam, S. & Siddiqui, A.M. (2010). Couette and Poiseuille Flows for Fourth-Grade Fluids using Optimal Homotopy Asymptotic Method. World Applied Science Journal, 9(1), 1228 – 1236.

Rivlin, R. & Ericksen J. L., (1995). Stress Deformation Relations for Isotropic Materials. Journal of Rational Mechanic, 4, 323 – 425.

Sander, C. J. & Holman, J. P. (1972). Franz Grashof and the Grashof Number. Int. J. Heat Mass Transfer, 15(2), 562 – 563.

Santhosha, B., Younus, S., Kamala, G. & Ramana, M. MV. (2017). Radiation and Chemical Reaction Effects on MHD Free Convective Heat and Mass Transfer Flow of a Viscoselastic Fluid Past a Porous Plate with Heat Generation/Absorption. International Journal of Chemical Sciences, 15(3), 1 – 16.

Satya, P. V., Venkateswarlu, B. & Devika, B. (2015). Chemical Reaction and Heat Source Effect on MHD Oscillatory Flow in an Irregular Channel. Ain Shams Engineering Journal, 1 – 10.

Shehzad, N., Zeeshan, A., & Ellahi, R. (2018). Electroosmotic Flow of MHD Power Law Al_2O_3-PVC Nano Fluid in a horizontal Channel: Couette – Poiseuille Flow Model .Commu. Theor. Phys., 65, 655 – 666.

Taha, A., Magan, A. B., & Mahomed, F. M. (2015). Invariant Solutions for the Unsteady Magnetohydrodynamics (MHD) Flow of a Fourth-Grade Fluid Induced due to the Impulsive Motion of a Flat Porous Plate. Brazil Journal of Physics, 45, 120 – 131.

Taza, G., Fazle, G., Islam, S., Shah, R. A., Khan, I., Nasir, S. & Sharidan (2016). Unsteady Thin Film Flow of a Fourth-Grade Fluid over a Vertical Moving and Oscillating Belt. Propulsion and Power Research, 5(3), 223 – 235.

Truesdell, C. & Noll, W. (1992). The Nonlinear Field Theories of Mechanics. Springer, 2.

Truesdell, C. & Noll, W. (2004). The Nonlinear Field Theories of Mechanics. 3rd edition, Springer, USA.

Umavathi, J. C., Liu, I. C. & Meera, S. (2010). Unsteady Flow and Heat Transfer of Porous Media Sandwiched between Viscous Fluids. Applied Mathematics and Mechanics, 31(12), 1497 – 1516.

Wang, Y. & Wu, W. (2007). Unsteady Flow of a Fourth-Grade Fluid due to an Oscillation Plate. Int. J. Nonlinear Mech., (42), 432 – 441.

Zaman, H., Shah, M. A. & Ibrahim, M. (2013). Unsteady Incompressible Couette Flow Problem for the Eyring – Powell Model with Porous Walls. American Journal of Computational Mathematics, 3, 313 – 325.

Zaman, H., Abbas, T., Sohail, A. & Ali, A. (2014). Couette Flow Problem for an Unsteady MHD Fourth-Grade Fluid with Hall Currents. Journal of Applied Mathematics and Physics, 2, 1 – 10.